Da Vinci and the Divine Proportion in Art Composition

July 7, 2014 by Gary Meisner 10 Comments Leonardo Da Vinci has long been associated with the golden ratio. This association was reinforced in popular culture in 2003 by Dan Brown's best selling book "The Da Vinci Code." The plot has pivotal clues involving the golden ratio and Fibonacci series. In 2006, the public awareness of the association grew when the book was turned into a movie starring veteran actor Tom Hanks. Da Vinci's association with the golden ratio, known in his time as the

Divine proportion, runs much longer and deeper.

Da Vinci's illustrations appear in Pacioli's book "The Divine Proportion"

Da Vinci created the illustrations for the book "De Divina Proportione" (The Divine Proportion) by Luca Pacioli. It was written in about 1497 and first published in 1509. Pacioli was a contemporary of Da Vinci's, and the book contains dozens of beautiful illustrations of three-dimensional geometric solids and templates for script letters in calligraphy. The original manuscript can be viewed online at http://issuu.com/s.c.williamslibrary/docs/de_divina_proportione. As of this writing in 2014, no

English translation is available.

In the book, Pacioli writes of mathematical and artistic proportion, especially the mathematics of the golden ratio and its application in art and architecture. Some geometric solids, such as dodecahedrons and icosahedrons, have inherent golden ratios in their dimensions and spatial positions of their intersecting lines. Other examples of golden ratios in the illustrations include the one architectural illustration in the book and the one script letter (G) that is not divided horizontally at its midpoint.

On the first page of De Divina Proportione, Pacioli states that his intent is to reveal to artists the secret of harmonic forms through the use of the Divine proportion, describing his writing as:

A work necessary for all the clear-sighted and inquiring human minds, in which everyone who loves to study philosophy, perspective, painting, sculpture, architecture, music and other mathematical disciplines will find a very delicate, subtle and admirable teaching and will delight in diverse questions touching on a very secret science.

Golden ratios / Divine proportions appear in Renaissance art in paintings of religious importance

Da Vinci's use of the Divine proportion is evident in some of his own works, even before his collaboration with Pacioli. The paintings I reviewed suggest that the use of the Divine proportion in paintings among Renaissance artists may have been more common in paintings of special religious significance. It appears that Da Vinci applied Divine proportions in his rendition of "The Annunciation, painted in about 1472-1473. An exact determination is difficult with 100% certainty because various images available of this painting are cropped slightly differently. It appears to be the basis for the dimensions of the walls and entry way of the courtyard, as well other elements of the composition, as shown in the photos below. Other golden ratios can be found, but to avoid any perception that this is arbitrary those shown are based on very distinct features of the painting. Botticelli also used the golden ration in several renditions he created of "The Annunciation" between 1485 and 1493. Michelangelo used it in "The Creation of Adam" in 1510.

Golden ratios based on width of painting canvas, using Florence Museum image. Note alignment of vertical walls and courtyard entry.

Golden ratios from left side to precise center of canvas, which aligns with the mountain peak. Note positioning of angle face and wings, as well as position of the one tree in the background that's different from all the others.

The Annunciation

Variation on this image shows divine proportion position of the entry to the courtyard

The ornamentation of the table is at the golden mean of its width

The peaks of the mountains in relation to the top of the canvas are in golden ratio proportion

Da Vinci's "The Last Supper" has many clear examples of Divine proportions

Perhaps one of the best illustrations of its use is in "The Last Supper," painted between 1494 and 1498. Various design and

architectural features show very clear golden ratios. Some believe that even the positions of the disciples around the table were placed in divine proportions to Jesus.

The Last Supper by Leonardo da Vinci

Last Supper detail showing golden ratios

Markings on the shield are golden ratios of its width

Small shield is a golden ratio of the width of its enclosure

From table top to ceiling, Jesus is at the midpoint and the top of the windows is at the golden ratio

Center of table is a golden ratio to its sides

Da Vinci's Mona Lisa and the Golden Ratio

One of Da Vinci's most famous paintings is "La Jaconde," the Mona Lisa. This painting was begun in about 1503 and work on it continued for years. It has similar composition to "Christ as Savior of the World." The application of the Divine proportion in this painting is the most subject to interpretation and debate. Unlike "The Last Supper" and "The Annunciation," the Mona Lisa has few straight lines or architectural elements to use as references points in making a determination. It's possible to find golden ratios in the Mona Lisa, but they're not as definitive. If you do a Google search for Mona Lisa golden ratio you'll also find some very creative interpretations of golden ratios in the Mona Lisa. Many of these are not easily supported as representing Da Vinci's intent because their starting points, position and size are somewhat arbitrary and inconsistent.

There are, however, observations that can be made about the Mona Lisa. The image on the left below shows that golden ratios from the sides of the canvas. This approach reveals no golden ratios from the canvas edges that align with key elements of the composition. There may still be golden ratios here though. The width of her face is very close to a golden ratio of the width of the canvas. This is illustrated by the yellow rectangle of the same dimensions. In the image on the right, we see that her eye is rather precisely aligned with the center of the canvas. Golden ratio lines from the center of the painting to the sides of the canvas align nicely with the width of her hair. There may also be golden ratios in the vertical dimensions of the painting. As with the painting of Christ above, the most prominent elements of the composition are her head, the garment neck line and her arm. These also show golden ratios in their positioning. So, while this is not as definitive as the straight lines of architectural elements in other paintings, a reasonable case can be made that the Mona Lisa also embodies intentional golden ratio proportions in its composition. Given Da Vinci's prior usage, it would not be unusual or unexpected for him to have applied it here as well.

Golden ratios from sides of canvas

Golden ratios from center of canvas

Analysis of Da Vinci's Vitruvian Man

Another of Da Vinci's most famous works is that of the Vitruvian Man, created around 1490. The official title of the drawing is "Le proporzioni del corpo umano secondo Vitruvio," or "The proportions of the human body according to Vitruvius." Wikipedia states,

The drawing is based on the correlations of ideal human

proportions with geometry described by the ancient Roman architect Vitruvius in Book III of his treatise De Architectura. Vitruvius described the human figure as being the principal source of proportion among the Classical orders of architecture. Vitruvius determined that the ideal body should be eight heads high. Leonardo's drawing is traditionally named in honor of the architect. This image demonstrates the blend of art and science during the Renaissance and provides the perfect example of Leonardo's deep understanding of proportion.

The Wikipedia article describes how Vitruvio measured the entire human body in integer fractions of the height of a man. Examination of the Vitruvian man illustration shows that the guide lines drawn by Da Vinci on the body appear to be based on integer fractions of the height, which is also equal to its width. The illustration below shows the height/width divided into 4ths, 5ths, 8ths and 10ths. This aligns with the guide lines drawn horizontally at the collar bone, chest, genitals and knee. They align horizontally with the guide lines drawn at the wrist, elbow and shoulder. Examination supports that Vitruvian Man was drawn to represent the fractional measures put forth by Vitruvius and is not based on the golden ratio. As discussed on the Great Pyramid page, however, irrational numbers like Phi can be closely approximated with integer fractions. So while Vitruvian Man may produce a very well proportioned human body, it seems unlikely that the human form would be based on a system of halfs, 4ths, 6ths, 7ths, 8ths and 10ths when we find constant rates of fractal expansion in the proportions of other living organisms. Other studies and approaches to measuring the human body express it just as well or better in golden ratio, which is better aligned to the growth relationships found in nature. See the Face, Body and Hand/Foot pages for illustrations.

The Vitruvian Man does have some dimensions that align with golden ratios. These are shown in the second drawing below and include the following points:

• In the distance from the Da Vinci's guide line drawn at the

hairline to the guide line at the foot, the following are all at golden ratio points:

the navel, which is most often associated with the golden ratio of the total height and not the height of the hairline the guidelines for the pectoral nipples the guidelines for the collar bone

 In the distance from the Da Vinci's guide line drawn at the elbow to the guideline at the fingertips

the base of the hand is at a golden ratio point.

Golden ratios are easy to identify and apply with simple tools

Divine proportions are quite easy for an artist to apply. All it takes is a simple two prong gauge that pivots at its golden ratio point. You take a measure on one side and then simply flip it around to get the golden ratio of that measure. Another popular design is the three prong gauge. In this case the golden ratio appears in a single line. Tools like this would provide very good estimates of golden ratio points, but are of course only as accurate as the precision of placement of their pivot points. They would not, in general, be as accurate as the pixel level analysis of those images as done in this article. This would explain some of the minor variations where the composition lines on a painting do not match the digital grid to the pixel. In most cases, however, the alignment is quite good and indicates the likeliness of the artist's intent to apply a golden ratio.

Not every Da Vinci's painting shows clear evidence of golden ratios

If you review all of Da Vinci's paintings, you will likely not find clear evidence of the golden ratio in many of them. He may have used it in many more paintings than those shown above. Many paintings though do not have distinct reference lines like those in these paintings, so it is difficult to support. While golden ratios may exist in elements of his other paintings, finding them after the fact can be subject to creative interpretation, and thus can be arbitrary and incorrect. I've tried in this article to provide examples where its application seems so clear that most reasonable people would see its intentional application by Da Vinci as verifiable and true. History has shown though that there will continue to be differences in viewpoints. If you have doubts, my recommendation is to do the analysis for yourself with high resolution images and a tool like PhiMatrix that creates golden ratios, custom ratios and rulers with pixel-level accuracy.