History

The phrase "equation of time" is derived from the medieval Latin, *aequātiō diērum* meaning "equation of days" or "difference of days". The word *aequātiō* was widely used in early astronomy to tabulate the difference between an observed value and the expected value (as in the equation of centre, the equation of the equinoxes, the equation of the epicycle). The difference between apparent solar time and mean time was recognized by astronomers since antiquity, but prior to the invention of accurate mechanical clocks in the mid-17th century, sundials were the only reliable timepieces, and solar time was the generally accepted standard.

A description of apparent and mean time was given by Nevil Maskelyne in the *Nautical Almanac* for 1767: "Apparent Time is that deduced immediately from the Sun, whether from the Observation of his passing the Meridian, or from his observed Rising or Setting. This Time is different from that shewn by Clocks and Watches well regulated at Land, which is called equated or mean Time." (He went on to say that, at sea, the apparent time found from observation of the sun must be corrected by the equation of time, if the observer requires the mean time.)[1]

The right time was essentially defined as that which was shown by a sundial. When good mechanical clocks were introduced they agreed with sundials only near four dates each year, so the equation of time was used to "correct" their readings to obtain sundial time. Some clocks, called equation clocks, included an internal mechanism to perform this "correction". Later, as clocks became the dominant good timepieces, uncorrected clock time i.e. "mean time" became the accepted standard. The readings of sundials, when they were used, were then, and often still are, corrected with the equation of time, used in the reverse direction from previously, to obtain clock time. Many sundials therefore have tables or graphs of the equation of time engraved on them to allow the user to make this correction.

The equation of time was used historically to set clocks. Between the invention of accurate clocks in 1656 and the advent of commercial time distribution services around 1900, there were three common land-based ways to set clocks. Firstly, in the unusual event of having an astronomer present, the sun's transit across the meridian (the moment the sun passed overhead) was noted, the clock was then set to noon and offset by the number of minutes given by the equation of time for that date. Secondly, and much more commonly, a sundial was read, a table of the equation of time (usually engraved on the dial), was consulted and the watch or clock set accordingly. These calculated the mean time, albeit local to a point of longitude. (The third method did not use the equation of time; instead, it used stellar observations to give sidereal time, exploiting the relationship between sidereal time and solar time.)[7]

Of course, the equation of time can still be used, when required, to obtain solar time from clock time. Devices such as solar trackers, which move to keep pace with the Sun's movements in the sky, frequently do not include sensors to determine the Sun's position. Instead, they are controlled by a clock mechanism, along with a mechanism that incorporates the equation of time to make the device keep pace with the Sun.

Ancient history — Babylon and Egypt[edit]

The irregular daily movement of the Sun was known by the Babylonians. Book III of Ptolemy's *Almagest* is primarily concerned with the Sun's anomaly, and he tabulated the equation of time in his *Handy Tables*.[8] Ptolemy discusses the correction needed to convert the meridian crossing of the Sun to mean solar time and takes into consideration the nonuniform motion of the Sun along the ecliptic and the meridian correction for the Sun's ecliptic longitude. He states the maximum correction is 8

⅓ time-degrees or 5⁄9 of an hour (Book III, chapter 9).[9] However he did not consider the effect to be relevant for most calculations since it was negligible for the slow-moving luminaries and only applied it for the fastest-moving luminary, the Moon.

Medieval and Renaissance astronomy[edit]

Based on Ptolemy's discussion in the Almagest, medieval Islamic astronomers such as al-Khwarizmi, al-Battani, Kushyar ibn Labban, al-Kashi and others, made improvements to the solar tables and the value of obliquity, and published tables of the equation of time (*taʿdīl al-ayyām bi layālayhā*) in their *zij* (astronomical tables).[10]

After that, the next substantial improvement in the computation didn't come until Kepler's final upset of the geocentric astronomy of the ancients. G. J. Toomer uses the medieval term "equation" from the

Latin *aequātiō*[n 1], for Ptolemy's difference between the mean solar time and the true solar time. Kepler's definition of the equation is "the difference between the number of degrees and minutes of the mean anomaly and the degrees and minutes of the corrected anomaly."[11]

Apparent time versus mean time[edit]

See also: Equation clock

Until the invention of the pendulum and the development of reliable clocks during the 17th century, the equation of time as defined by Ptolemy remained a curiosity, of importance only to astronomers. However, when mechanical clocks started to take over timekeeping from sundials, which had served humanity for centuries, the difference between clock time and solar time became an issue for everyday life. Apparent solar time (or true or real solar time) is the time indicated by the Sun on a sundial (or measured by its transit over a preferred local meridian), while *mean solar time* is the average as indicated by well-regulated clocks. The first tables to give the equation of time in an essentially correct way were published in 1665 by Christiaan Huygens.[12] Huygens, following the tradition of Ptolemy and medieval astronomers in general, set his values for the equation of time so as to make all values positive throughout the year.[13][n 2]

Another set of tables was published in 1672–73 by John Flamsteed who later became the first Astronomer Royal of the new Greenwich Observatory. These appear to have been the first essentially correct tables that gave today's meaning of Mean Time (rather than mean time based on the latest sunrise of the year as proposed by Huygens). Flamsteed adopted the convention of tabulating and naming the correction in the sense that it was to be applied to the apparent time to give mean time.[14]

The equation of time, correctly based on the two major components of the Sun's irregularity of apparent motion,[n 3] was not generally adopted until after Flamsteed's tables of 1672–73, published with the posthumous edition of the works of Jeremiah Horrocks.[15]

Robert Hooke (1635–1703), who mathematically analyzed the universal joint, was the first to note that the geometry and mathematical description of the (non-secular) equation of time and the universal joint were identical, and proposed the use of a universal joint in the construction of a "mechanical sundial".[16]

18th and early 19th centuries[edit]

The corrections in Flamsteed's tables of 1672–73 and 1680 gave mean time computed essentially correctly and without need for further offset. But the numerical values in tables of the equation of time have somewhat changed since then, owing to three factors:

- general improvements in accuracy that came from refinements in astronomical measurement techniques,
- slow intrinsic changes in the equation of time, occurring as a result of small long-term changes in the Earth's obliquity and eccentricity (affecting for instance the distance and dates of perihelion), and
- the inclusion of small sources of additional variation in the apparent motion of the Sun, unknown in the 17th century, but discovered from the 18th century onwards, including the effects of the Moon[n 4], Venus and Jupiter.[17]

A sundial made in 1812, by Whitehurst & Son with a circular scale showing the equation of time correction. This is now on display in the Derby Museum.

From 1767 to 1833, the British *Nautical Almanac and Astronomical Ephemeris* tabulated the equation of time in the sense 'mean minus apparent solar time'. Times in the Almanac were in apparent solar time, because time aboard ship was most often determined by observing the Sun. In the unusual case that the mean solar time of an observation was needed, one would apply the equation of time to *apparent* solar time. In the issues since 1834, all times have been in mean solar time, because by then the time aboard ship was increasingly often determined by marine chronometers. In the unusual case that the apparent solar time of an observation was needed, one would apply the equation of time to *mean* solar time, requiring all differences in the equation of time to have the opposite sign than before.

As the apparent daily movement of the Sun is one revolution per day, that is 360° every 24 hours, and the Sun itself appears as a disc of about 0.5° in the sky, simple sundials can be read to a maximum accuracy of about one minute. Since the equation of time has a range of about 33 minutes, the difference between sundial time and clock time cannot be ignored. In addition to the equation of time, one also has to apply corrections due to one's distance from the local time zone meridian and summer time, if any.

The tiny increase of the mean solar day itself due to the slowing down of the Earth's rotation, by about 2 ms per day per century, which currently accumulates up to about 1 second every year, is not taken into account in traditional definitions of the equation of time, as it is imperceptible at the accuracy level of sundials.

Explanations for the major components of the equation of time[edit]

Eccentricity of the Earth's orbit[edit]

Graph showing the equation of time (red solid line) along with its two main components plotted separately, the part due to the obliquity of the ecliptic (mauve dashed line) and the part due to the Sun's varying apparent speed along the ecliptic due to the eccentricity of the Earth's orbit (dark blue dash & dot line)

The Earth revolves around the Sun. As seen from Earth, the Sun appears to revolve once around the Earth through the background stars in one year. If the Earth orbited the Sun with a constant speed, in a circular orbit in a plane perpendicular to the Earth's axis, then the Sun would culminate every day at exactly the same time, and be a perfect time keeper (except for the very small effect of the slowing rotation of the Earth). But the orbit of the Earth is an ellipse not centered on the Sun, and its speed varies between 30.287 and 29.291 km/s, according to Kepler's laws of planetary motion, and its angular speed also varies, and thus the Sun appears to move faster (relative to the background stars) at perihelion (currently around 3 January) and slower at aphelion a half year later. At these extreme points this effect varies the real solar day by 7.9 s/day from its mean. Consequently, the smaller daily differences on other days in speed are cumulative until these points, reflecting how the planet accelerates and decelerates compared to the mean. As a result, the eccentricity of the Earth's orbit contributes a sine wave variation with an amplitude of 7.66 min and a period of one year to the equation of time. The zero points are reached at perihelion (at the beginning of January) and aphelion (beginning of July); the extreme values are in early April (negative) and early October (positive).

Obliquity of the ecliptic